LambdaFM: Learning Optimal Ranking with Factorization Machines Using Lambda Surrogates

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ABSTRACT

State-of-the-art item recommendation algorithms, which apply Factorization Machines (FM) as a scoring function and pairwise ranking loss as a trainer (PRFM for short), have been recently investigated for the implicit feedback based context-aware recommendation problem (IFCAR). However, good recommenders particularly emphasize on the accuracy near the top of the ranked list, and typical pairwise loss functions might not match well with such a requirement. In this paper, we demonstrate, both theoretically and empirically, PRFM models usually lead to non-optimal item recommendation results due to such a mismatch. Inspired by the success of LambdaRank, we introduce Lambda Factorization Machines (LambdaFM), which is particularly intended for optimizing ranking performance for IFCAR. We also point out that the original lambda function suffers from the issue of expensive computational complexity in such settings due to a large amount of unobserved feedback. Hence, instead of directly adopting the original lambda strategy, we create three effective lambda surrogates by conducting a theoretical analysis for lambda from the top-N optimization perspective. Further, we prove that the proposed lambda surrogates are generic and applicable to a large set of pairwise ranking loss functions. Experimental results demonstrate LambdaFM significantly outperforms state-of-the-art algorithms on three real-world datasets in terms of four standard ranking measures.

Keywords

Factorization Machines; PRFM; LambdaFM; Context-aware; Pairwise Ranking; Top-N Recommendation

1. INTRODUCTION

Context-aware recommender systems exploit contextual information such as time, location or social connections to personalize item recommendation for users. For instance, in a music recommender like www.last.fm, road situation or driver's mood may be important contextual factors to consider when suggesting a music track in a car [1]. To leverage the context which users or items are associated with, several effective models have been proposed, among which Factorization Machines (FM) [21] gain much popularity thanks to its elegant theory in seamless integration of sparse context and exploration of context interactions. FM is originally developed for rating prediction task [6], which is based on explicit user feedback. However, in practice most observed feedback is implicit [23], such as clicks, check-ins, purchases, music playing, and thus it is much more readily available and pervasive. Besides, the goal of recommender systems is to find a few specific and ordered items which are supposed to be the most appealing ones to the users, known as the top-N item recommendation task [6]. Previous literature has pointed out that algorithms optimized for rating prediction do not translate into accuracy improvement in terms of item recommendation [6]. In this paper, we study the problem of optimizing item ranking in implicit feedback based context-aware recommendation problem (IFCAR). To address both context-aware and implicit feedback scenarios, the ranking based FM algorithm by combining pairwise learning to rank (LtR) techniques (PRFM [21, 18]) has been recently investigated. The main benefit of PRFM is the capability to model the interactions between context features under huge sparsity, where typical LtR approaches usually fail.

To our knowledge, PRFM works indeed much better than FM in the settings of IFCAR. Nevertheless, it is reasonable to argue that pairwise learning is position-independent: an incorrect pairwise-wise ordering at the bottom of the list impacts the score just as much as that at the top of the list [14]. However, for top-N item recommendation task, the learning quality is highly position-dependent: the higher accuracy at the top of the list is more important to the recommendation quality than that at the low-position items, reflected in the rank biased metrics such as NDCG [16] and MRR [26]. Hence, pairwise loss might still be a suboptimal scheme for ranking tasks. In this work, we conduct a detailed analysis from the top-N optimization perspective, and shed light on how PRFM results in non-optimal ranking in the setting of IFCAR. Besides the theoretical analysis, we also provide the experimental results to reveal the inconsistency between the pairwise classification evaluation metric and standard ranking metrics (e.g., AUC [23] vs. MRR).

Inheriting the idea of differentiating pairs in LambdaRank [19], we propose LambdaFM, an advanced variant of PRFM by directly optimizing the rank biased metrics. We point out that the original lambda function [19] is computationally in-

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tractable due to a large number of unobserved feedback in IFCAR settings. To tackle such a problem, we implement three alternative surrogates based on the analysis of lambda. Furthermore, we claim that the proposed lambda surrogates are more general and can be applied to a large class of pairwise loss functions. By applying such loss functions, a family of PRFM and LambdaFM algorithms have been developed. Finally, we perform thorough experiments on three real-world datasets and compare LambdaFM with state-of-the-art approaches. Our results show that LambdaFM noticeably outperforms all counterparts in terms of four standard ranking evaluation metrics.

2. RELATED WORK

The approach presented in this work is rooted in research areas of context-aware recommendation and Learning to Rank (LtR) techniques. Hence, we discuss the most relevant previous contribution in each of the two areas and position our work with respect to them.

Context-aware Recommender Systems. Researchers have devoted a lot of efforts to context-aware recommender systems (CARS). Early work in CARS performed pre- or post-filtering of the input data to make standard methods context-aware, and thus ignored the potential interactions between different context variables. Recent research mostly focuses on integrating context into factorization models. Two lines of contributions have been presented to date, one based on tensor factorization (TF) [11] and the other on Factorization Machines (FM) [21] as well as its variant SVDFeature [4]. The type of context by TF is usually limited to categorical variables. By contrast, the interactions of context by FM are more general, i.e., not limited to categorical ones. On the other hand, both approaches are originally designed for the rating prediction task [6], which are based on explicit user feedback. However, in most real-world scenarios, only implicit user behavior is observed and there is no explicit rating [22, 23]. Besides, the goal of item recommendation is preferred as a ranking task rather than a rating prediction one. The state-of-the-art methods by combining LtR and context-aware models (e.g., TF and FM) provide promising solutions [24, 25, 18]. For example, TFMAP [25] utilizes TF to model the three-way user-item-context relations, and the factorization model is learned by directly optimizing Mean Average Precision (MAP); similar work by CARS2 [24] attempts both pairwise and listwise loss functions to learn a novel TF model; PRFM [21, 18], on the other side, applies FM as the ranking function to model the pairwise interactions of context, and optimizes FM by maximizing the AUC metric.

LtR. There are two major approaches in the field of LtR, namely pairwise [2, 23, 7, 16] and listwise approaches [3, 26, 25]. In the pairwise settings, LtR problem is approximated by a classification problem, and thus existing methods in classification can be directly applied. However, it has been pointed out in previous Information Retrieval (IR) literature that pairwise approaches are designed to minimize the classification errors of objective pairs, rather than errors in ranking of items [3]. In other words, the pairwise loss does not inversely correlate with the ranking measures such as Normalized Discounted Cumulative Gain (NDCG) [16] and MAP [25]. By contrast, listwise approaches solve this problem in a more elegant way where the models are formalized to directly optimize a specific list-level ranking metric. Generally, it is non-trivial to directly optimize the ranking performance measures because they are often not continuous or differentiable, e.g., NDCG, MAP and MRR. Moreover, due to the sparsity characteristic in IFCAR settings, listwise approaches fail to make use of unobserved feedback, which empirically leads to non-improved or even worse performance than pairwise ones [24]. The other way is to add listwise information into pairwise learning. The most typical work is LambdaRank [19], where the change of NDCG of the ranking list if switching the item pair is incorporated into the pairwise loss in order to reshape the model by emphasizing its learning over the item pairs leading to large NCDG drop.

It is worth noticing that previous LtR models (e.g. Ranking SVM [9], RankBoost [7], RankNet [2], ListNet [3], LambdaRank [19]) were originally proposed for IR tasks with dense features, which might not be directly applied in recommender systems (RS) with huge sparse context feature space. Besides, RS target at personalization, which means each user should attain one set of parameters for personalized ranking, whereas the conventional LtR normally learns one set of global parameters [3], i.e., non-personalization. Hence, in this paper we build our contributions on the state-of-the-art algorithm of PRFM. By a detailed analysis for the ranking performance of PRFM, we present LambdaFM motivated by the idea of LambdaRank. However, computing such a lambda poses an efficiency challenge in learning the model. By analyzing the function of lambda, we present three alternative surrogate strategies that are capable of achieving equivalent performance.

3. PRELIMINARIES

In this section, we first recapitulate the idea and implementation of PRFM based on the pairwise cross entropy loss. Then we show that PRFM suffers from a suboptimal ranking for top-N item recommendations. Motivated by this, we devise the LambdaFM algorithm by applying the idea from LambdaRank.

3.1 Pairwise Ranking Factorization Machines

In the context of recommendation, let U be the whole set of users and I the whole set of items. Assume that the learning algorithm is given a set of pairs of items $(i, j) \in I$ for user $u \in U$, together with the desired target value $\overline{P}_{ij}^u$ for the posterior probability, and let $\overline{P}_{ij}^u \in \{0, 0.5, 1\}$ be defined as 1 if i is preferred over item j by user u, 0 if item i is less preferred, and 0.5 if they are given the same preference. The cross entropy (CE) [2] loss is defined as

$$L = \sum_{u \in U} \sum_{i \in I} \sum_{j \in I} -\overline{P}_{ij}^u \log P_{ij}^u - \left(1 - \overline{P}_{ij}^u\right) \log\left(1 - P_{ij}^u\right) \quad (1)$$

where P_{ij}^u is the modeled probability

$$P_{ij}^u = \frac{1}{1 + \exp\left(-\sigma\left(\hat{y}(\boldsymbol{x}^i) - \hat{y}(\boldsymbol{x}^j)\right)\right)} \quad (2)$$

where σ determines the shape of sigmoid with 1 as default value, $\boldsymbol{x} \in \mathbb{R}^n$ denotes the input vector, and $\hat{y}(\boldsymbol{x})$ is the ranking score computed by 2-order FM

$$\hat{y}(\boldsymbol{x}) = w_0 + \sum_{k=1}^{n} w_k x_k + \frac{1}{2} \sum_{f=1}^{d} \left(\big(\sum_{k=1}^{n} v_{k,f} x_k \big)^2 - \sum_{k=1}^{n} v_{k,f}^2 x_k^2 \right) \quad (3)$$

where n is the number of context variables, d is a hyper-parameter that denotes the dimensionality of latent factors, and w_0, w_k, $v_{k,f}$ are the model parameters to be esti-

Algorithm 1 Ranking FM Learning

Input: Training dataset, regularization parameters γ_θ, learning rate η
Output: Parameters $\Theta = (\boldsymbol{w}, \boldsymbol{V})$
Initialize Θ: $\boldsymbol{w} \leftarrow (0, ..., 0)$; $\boldsymbol{V} \sim \mathcal{N}(0, 0.1)$;
repeat
  Uniformly draw u from U ;
  Uniformly draw i from I_u ;
  Uniformly draw j from $I \backslash I_u$;
  for $k \in \{1, ..., n\} \wedge x_k \neq 0$ **do**
    Update w_k as in Eq. (9)
  end for
  for $f \in \{1, ..., d\}$ **do**
    for $k \in \{1, ..., n\} \wedge x_k \neq 0$ **do**
      Update $v_{k,f}$ as in Eq. (10)
    end for
  end for
until convergence
return Θ

mated, i.e., $\Theta = \{w_0, w_1, ..., w_n, v_{1,1}, ..., v_{n,d}\} = \{w_0, \boldsymbol{w}, \boldsymbol{V}\}$[1]. As previously mentioned, we focus on the recommendation problem in IFCAR settings, where only implicit feedback is available. To simplify Eq. (1), we formalize implicit feedback training data for user u as

$$D_u = \{\langle i, j \rangle_u | i \in I_u \wedge j \in I \backslash I_u\} \tag{4}$$

where I_u represents the set of items that user u have given a positive feedback, i and j are an observed and unknown item for u, respectively. Thus, we have $\overline{P}_{ij}^u$=1. Now the CE loss function and gradient for each $\langle i, j \rangle_u$ pair in our settings becomes

$$L\big(\langle i, j \rangle_u\big) = \log\left(1 + \exp\Big(-\sigma\big(\hat{y}(\boldsymbol{x}^i) - \hat{y}(\boldsymbol{x}^j)\big)\Big)\right) \tag{5}$$

$$\frac{\partial L(\langle i, j \rangle_u)}{\partial \theta} = \lambda_{i,j}\left(\frac{\partial \hat{y}(\boldsymbol{x}^i)}{\partial \theta} - \frac{\partial \hat{y}(\boldsymbol{x}^j)}{\partial \theta}\right) \tag{6}$$

where $\lambda_{i,j}$ is the learning weight[2] for $\langle i, j \rangle_u$ pair and is defined as

$$\lambda_{i,j} = \frac{\partial L(\langle i, j \rangle_u)}{\partial\big(\hat{y}(\boldsymbol{x}^i) - \hat{y}(\boldsymbol{x}^j)\big)} = -\frac{\sigma}{1 + \exp\Big(\sigma\big(\hat{y}(\boldsymbol{x}^i) - \hat{y}(\boldsymbol{x}^j)\big)\Big)} \tag{7}$$

According to the property of Multilinearity [21]. The gradient of Eq. (3) can be derived

$$\frac{\partial \hat{y}(\boldsymbol{x}^i)}{\partial \theta} = \begin{cases} x_k^i & \text{if } \theta \text{ is } w_k \\ x_k^i \sum_{l=1}^n v_{l,f} x_l^i - v_{k,f} {x_k^i}^2 & \text{if } \theta \text{ is } v_{k,f} \end{cases} \tag{8}$$

By combining Eqs. (5)-(8), we obtain

$$w_k \leftarrow w_k - \eta\Big(\lambda_{i,j}(x_k^i - x_k^j) + \gamma_{w_k} w_k\Big) \tag{9}$$

$$\begin{aligned} v_{k,f} \leftarrow v_{k,f} - \eta\Big(\lambda_{i,j}\big(\sum_{l=1}^n v_{l,f}(x_k^i x_l^i - x_k^j x_l^j) \\ - v_{k,f}({x_k^i}^2 - {x_k^j}^2)\big) + \gamma_{v_{k,f}} v_{k,f}\Big) \end{aligned} \tag{10}$$

where γ_θ (i.e., γ_{w_k}, $\gamma_{v_{k,f}}$) is a hyper-parameter for the L2 regularization. To handle the large number of unobserved feedback (i.e., $I \backslash I_u$), the common practice is to apply Stochastic Gradient Descent (SGD) with bootstrapping [23]. Finally, the pseudo code of PRFM in the settings of IFCAR is shown in Algorithm 1.

[1] In the remainder, scalar variables are set in the default math font, e.g., w_0, w_k, $v_{k,f}$, while vectors (lower case) and matrices (upper case) are in bold face, e.g., $\boldsymbol{w}$, $\boldsymbol{V}$.

[2] $\lambda_{i,j}$ can be read as how much influence $\langle i, j \rangle_u$ has for updating Θ.

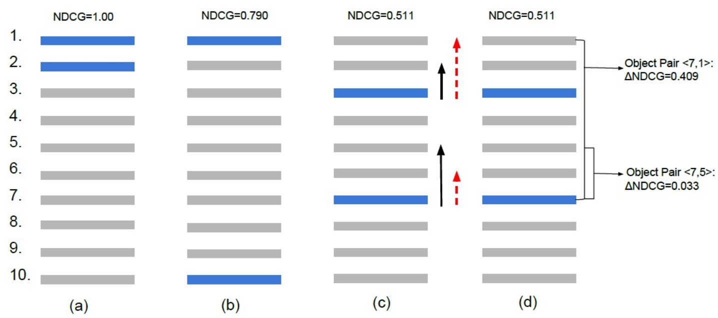


Figure 1: A set of items ordered for a given user using a binary relevance measure. The blue bars represent the items preferred by the user, while the light gray bars are those not preferred by the user. (a) is the ideal ranking; (b) is a ranking with eight pairwise errors; (c) and (d) are a ranking with seven pairwise errors by moving the top items of (b) down two rank levels, and the bottom preferred items up three. The two arrows (black solid and red dashed) of (c) denote two ways to minimize the pairwise errors. (d) shows the change in NDCG by swapping the orders of two items (e.g., item 7 and item 1).

3.2 Lambda Motivation

Top-N item recommendation is often referred to as a ranking task, where ranking measures like Normalized Discounted Cumulative Gain (NDCG) are widely adopted to evaluate the recommendation accuracy. However, it has been pointed out in previous IR literature (e.g., [19]) that pairwise loss functions might not match well with these measures. For easy reading, we start to state the problem by an intuitive example in the form of implicit feedback.

Figure 1 is a schematic that illustrates the relations of pairwise loss and NDCG. By comparison of (b) and (c), we observe two mismatch between them. First, the pairwise errors decrease from eight (b) to seven (c), along with the value of NDCG decreasing from 0.790 to 0.511. However, an ideal recommender model is supposed to increase the NDCG value with the drop of pairwise errors. Second, in (c), the arrows denote the next optimization direction and strength, and thus can be regarded as the gradients of PRFM. Although both arrows reduce the same amount of pairwise errors (i.e., three), the red dashed arrows lead to more gains as desired. This is because the new NDCGs, after the optimization, are 0.624 and 0.832 by the black solid and red dashed arrows, respectively.

According to the above counterexamples, we clearly see that the pairwise loss function used in the optimization process is not a desired one as reducing its value does not necessarily increase the ranking performance. This means, PRFM might still be a suboptimal scheme for top-N recommendations. This motivates us to study whether PRFM can be improved by concentrating on maximizing the ranking measures.

To overcome the above challenge, Lambda-based methods [19] in the field of IR have been proposed by directly optimizing the ranking biased performance measures. Inheriting the idea of LambdaRank, we design a new recommendation model named Lambda Factorization Machines (LambdaFM), an extended version of PRFM for solving top-N context-aware recommendations in implicit feedback domains. Following this idea, we can design a similar lambda function as $f(\lambda_{i,j}, \zeta_u)$, where ζ_u is the current item ranking list for user u (under context c). With NDCG as target, $f(\lambda_{i,j}, \zeta_u)$ is given

$$f(\lambda_{i,j}, \zeta_u) = \lambda_{i,j} |\triangle NDCG_{ij}| \tag{11}$$

where $\triangle NDCG_{ij}$ is the NDCG difference of the ranking list for a specific user if the positions (ranks) of items i, j get switched. LambdaFM can be implemented by replacing $\lambda_{i,j}$ with $f(\lambda_{i,j}, \zeta_u)$ in Eqs. (9) and (10)[3].

We find that the above implementation is reasonable for multi-label learning tasks in typical IR tasks, but not applicable into IFCAR settings. This is because to calculate $\triangle NDCG_{ij}$ it requires to compute scores of all items to obtain the rankings of item i and j. For the traditional LtR, the candidate URLs returned for a given query in training datasets have usually been limited to a small size [31]. However, for recommendation, since there is no query at all, all unobserved (non-positive) items should be regarded as candidates, which leads to a very large size of candidates (see the items column in Table 1). Thus the complexity to calculate each training pair in IFCAR settings becomes $O(|I| \cdot T_{pred})$, where T_{pred} is the time for predicting a score by Eq. (3). That is, the original lambda function devised for LambdaRank is impractical for our settings.

4. LAMBDA STRATEGIES

To handle the above problem, we need to revisit Eqs. (11), from which $\triangle NDCG_{ij}$ can be interpreted as a weight function that it rewards a training pair by raising the learning weight (i.e., $\lambda_{i,j}$) if the difference of NDCG is larger after swapping the two items, otherwise it penalizes the training pair by shrinking the learning weight. Suppose an ideal lambda function $f(\lambda_{i,j}, \zeta_u)$ for each training pair $\langle i, j \rangle_u$ is given with the current item ranking list ζ_u, if we design a scheme that generates the training item pair $\langle i, j \rangle_u$ with the probability proportional to $f(\lambda_{i,j}, \zeta_u)/\lambda_{i,j}$ (just like $\triangle NDCG_{ij}$ in Eqs. (11)), then we are able to construct an almost equivalent training model. In other words, a higher proportion of item pairs should be drawn according to the probability distribution $p_j \propto f(\lambda_{i,j}, \zeta_u)/\lambda_{i,j}$ if they have a larger $\triangle NDCG$ by swapping.

On the other hand, we observe that item pairs with a larger $\triangle NDCG$ contribute more to the desired loss and lead to a larger NDCG value. In the following, we refer to a training pair $\langle i, j \rangle_u$ as an informative item pair, if $\triangle NDCG_{ij}$ is larger after swapping i and j than another pair $\langle i, j' \rangle$. The unobserved item j is called a good or informative item. Thus two research questions arise: (1) how to judge which item pairs are more informative? and (2) how to pick up informative items? Figure 1 (d) gives an example of which item pairs should have a higher weight. Obviously, we observe that the quantity of $\triangle NDCG_{71}$ is larger than that of $\triangle NDCG_{75}$. This indicates $\triangle NDCG_{ij}$ is likely to be larger if unobserved (e.g., unselected) items have a higher rank (or a relatively higher score by Eq. (3)). This is intuitively correct as the high ranked non-positive items hurt the ranking performance (users' feelings) more than the low ranked ones. The other side of the intuition is that low ranked positive items contribute similarly as high ranked non-positive items during the training process, which would be provided with more details later.

[3]Due to space limitations, we leave out more details and refer the interested readers to [29, 19] for the technical part.

In the following, we provide three intuitive lambda surrogates for addressing the two research questions and refer to PRFM with suggested lambda surrogates as LambdaFM (LFM for short in some places).

4.1 Static & Context-independent Sampler

We believe that a popular item $j \in I \backslash I_u$ is supposed to be a reasonable substitute for a high ranked non-positive item, i.e., the so-called informative item. The reason is intuitive as an ideal learning algorithm is expected to restore the pairwise ordering relations and rank most positive items with larger ranking scores than non-positive ones. As we know, the more popular an item is, the more times it acts as a positive one. Besides, popular items are more likely to be suggested by a recommender in general, and thus they have higher chances to be positive items. That is, the observation that a user prefers an item over a popular one provides more information to capture her potential interest than that she prefers an item over a very unpopular one. Thus, it is reasonable to assign a higher weight to non-positive items with high popularity, or sampling such items with a higher probability.

In fact, it has been recognized that item popularity distribution in most real-world recommendation datasets has a heavy tail [17, 13], following an approximate power-law or exponential distribution[4]. Accordingly, most non-positive items drawn by uniform sampling in Algorithm 1 are unpopular due to the long tail distribution, and thus contribute less to the desired loss function. Based on the analysis, it is reasonable to present a popularity-aware sampler to replace the uniform one before performing each SGD. Let p_j denote the sampling distribution for item j. In this work, we draw unobserved items with the probability proportional to the empirical popularity distribution, e.g., an exponential distribution (In practice, the distribution p_j can be replaced with other analytic distributions, such as geometric and linear distributions, or non-parametric ones).

$$p_j \propto \exp\Big(-\frac{r(j)+1}{|I| \times \rho}\Big), \rho \in (0, 1] \tag{12}$$

where $r(j)$ represents the rank of item j among all items I according to the overall popularity, ρ is the parameter of the distribution. Therefore, Line 7 in Algorithm 1 can be directly replaced as the above sampler, i.e., Eq. (12). Hereafter, we denote PRFM with the static popularity-aware sampler as LFM-S.

The proposed sampler has three important properties:

- *Static.* The sampling procedure is static, i.e., the distribution does not change during the training process.
- *Context-Invariant.* The item popularity is independent of context information according to Eq. (12).
- *Efficient.* The sampling strategy does not increase the computational complexity since the popularity distribution is static and thus can be calculated in advance.

4.2 Dynamic & Context-aware Sampler

The second sampler is a dynamic one which is able to change the sampling procedure while the model parameters Θ are updated. The main difference of dynamic sampling is that the item rank is computed by their scores instead of global popularity. As it is computationally expensive to

[4]We have examined the long tail property of our example datasets, while details are omitted for space reasons.

Algorithm 2 Rank-Aware Dynamic Sampling

1: **Require:** Unobserved item set $I \backslash I_u$, scoring function $\hat{y}(\cdot)$, parameter m, ρ
2: Draw sample $j_1,...,j_m$ uniformly from $I \backslash I_u$
3: Compute $\hat{y}(\boldsymbol{x}^{j_1}),...,\hat{y}(\boldsymbol{x}^{j_m})$
4: Sort $j_1,...,j_m$ by descending order, i.e., $r(j_m) \propto \frac{1}{\hat{y}(\boldsymbol{x}^{j_m})}$
5: Return one item from the sorted list with the exponential distribution $p_j \propto \exp(-(r(j)+1)/(m \times \rho))$.

calculate all item scores in the item list given a user and context, which is different from the static item popularity calculated in advance, we first perform a uniform sampling to obtain m candidates. Then we calculate the candidate scores and sample the candidates also by an exponential distribution. As the first sampling is uniform, the sampling probability density for each item is equivalent with that from the original (costy) global sampling. A straightforward algorithm can be implemented by Algorithm 2. We denote PRFM with the dynamic sampler as LFM-D. It can be seen the sampling procedure is dynamic and context-aware.

- The sampler selects non-positive items dynamically according to the current item ranks which are likely to be different at each update.
- The item rank is computed by the FM function (i.e., Eq. (3)), which is clearly context-aware.
- The complexity for parameter update of each training pair is $O(mT_{\text{pred}}$ (Line 3)$+m \log m$ (Line 4)), where the number of sampling items m can be set to a small value (e.g., m=10, 20, 50). Thus introducing the dynamic sampler will not increase the time complexity much.

4.3 Rank-aware Weighted Approximation

The above two surrogates are essentially based on non-positive item sampling techniques, the core idea is to push non-positive items with higher ranks down from the top positions. Based on the same intuition, an equivalent way is to pull positive items with lower ranks up from the bottom positions. That is, we have to place less emphasis on the highly ranked positives and more emphasis on the lowly ranked ones. Specifically, if a positive item is ranked top in the list, then we use a reweighting term $\Gamma\big(r(i)\big)$ to assign a smaller weight to the learning weight $\lambda_{i,j}$ such that it will not cost the loss too much. However, if a positive item is not ranked at top positions, $\Gamma\big(r(i)\big)$ will assign a larger weight to the gradient, pushing the positive item to the top.

$$f(\lambda_{i,j}, \zeta_u) = \Gamma\big(r(i)\big)\lambda_{i,j} \tag{13}$$

By considering maximizing objectives e.g., reciprocal rank, we define $\Gamma\big(r(i)\big)$ as

$$\Gamma\big(r(i)\big) = \frac{1}{\Gamma\big(I\big)} \sum_{r=0}^{r(i)} \frac{1}{r+1} \tag{14}$$

where $\Gamma\big(I\big)$ is the normalization term calculated by the theoretically lowest position of a positive item, i.e.,

$$\Gamma\big(I\big) = \sum_{r \in I} \frac{1}{r+1} \tag{15}$$

However, the same issue occurs again, i.e., the rank of the positive item i is unknown without computing the scores of all items. In contrast with non-positive item sampling methods, sampling positives is clearly useless as there are only a small fraction of positive items (i.e., $|I_u|$), and thus all of them should be utilized to alleviate the sparsity. Interestingly, we observe that it is possible to compute the approximate rank of positive item i by sampling one incorrectly-ranked item. More specifically, given a $\langle u, i \rangle$ pair, we repeatedly draw an item from I until we obtain an incorrectly-ranked item j such that $\hat{y}(\boldsymbol{x}^i) - \hat{y}(\boldsymbol{x}^j) \leq \varepsilon$ and $\overline{P}_{ij}^u$=1, where ε is a positive margin value[5]. Let T denote the size of sampling trials before obtaining such an item. Apparently, the sampling process corresponds to a geometric distribution with parameter $p = \frac{r(i)}{|I|-1}$. Since the number of sampling trials can be regarded as the expectation of parameter p, we have $T \approx \lceil \frac{1}{p} \rceil = \lceil \frac{|I|-1}{r(i)} \rceil$, where $\lceil \cdot \rceil$ is the ceiling function. Our idea here is similar to that used in [27, 28] for a different problem. Finally, by using the estimation, we rewrite the gradient

$$\frac{\partial L(\langle i,j \rangle_u)}{\partial \theta} = \frac{\sum_{r=0}^{\lceil \frac{|I|-1}{T} \rceil} \frac{1}{r+1}}{\Gamma\big(I\big)} \lambda_{i,j} \left(\frac{\partial \hat{y}(\boldsymbol{x}^i)}{\partial \theta} - \frac{\partial \hat{y}(\boldsymbol{x}^j)}{\partial \theta} \right) \tag{16}$$

Algorithm 3 LFM-W Learning

Input: Training dataset, regularization parameters γ, learning rate η
Output: Parameters $\Theta = (\boldsymbol{w}, \boldsymbol{V})$
Initialize Θ: $\boldsymbol{w} \leftarrow (0, ..., 0)$; $\boldsymbol{V} \sim \mathcal{N}(0, 0.1)$;
repeat
  Uniformly draw u from U
  Uniformly draw i from I_u
  Calculate $\hat{y}(\boldsymbol{x}^i)$
  Set T=0
  do
    Uniformly draw j from I
    Calculate $\hat{y}(\boldsymbol{x}^j)$
    T+=1
  while $\big(\hat{y}(\boldsymbol{x}^i) - \hat{y}(\boldsymbol{x}^j) > \varepsilon \parallel \overline{P}_{ij}^u \neq 1\big) \wedge \big(T < |I| - 1\ \big)$
  if $\hat{y}(\boldsymbol{x}^i) - \hat{y}(\boldsymbol{x}^j) \leq \varepsilon \wedge \overline{P}_{ij}^u = 1$ **then**
    Calculate $\lambda_{i,j}, \Gamma\big(I\big)$ according to Eqs. (7) and (15)
    for $k \in \{1, ..., n\} \wedge x_k \neq 0$ **do**
      Update w_k:
      $$w_k \leftarrow w_k - \eta \Big(\lambda_{i,j} (x_k^i - x_k^j) \frac{\sum_{r=0}^{\lceil \frac{|I|-1}{T} \rceil} \frac{1}{r+1}}{\Gamma\big(I\big)} - \gamma_{w_k} w_k \Big)$$
    end for
    for $f \in \{1, ..., d\}$ **do**
      for $k \in \{1, ..., n\} \wedge x_k \neq 0$ **do**
        Update $v_{k,f}$:
        $$v_{k,f} \leftarrow v_{k,f} - \eta \Big(\lambda_{i,j} \big(\sum_{l=1}^{n} v_{l,f} (x_k^i x_l^i - x_k^j x_l^j) - v_{k,f} ({x_k^i}^2 - {x_k^j}^2) \big) \frac{\sum_{r=0}^{\lceil \frac{|I|-1}{T} \rceil} \frac{1}{r+1}}{\Gamma\big(I\big)} - \gamma_{v_{k,f}} v_{k,f} \Big)$$
      end for
    end for
  end if
until convergence
return Θ

We denote the proposed algorithm based weighted approximation as LFM-W in Algorithm 3. In this algorithm, we iterate through all positive items $i \in I_u$ for each user and update the model parameters $\boldsymbol{w}, \boldsymbol{V}$ until the procedure converges. In each iteration, given a user-item pair, the sampling process is first performed so as to estimate violating

[5]We hypothesize that item j is ranked higher than i for user u only if $\hat{y}(\boldsymbol{x}^i) - \hat{y}(\boldsymbol{x}^j) \leq \varepsilon$, the default value of ε in this paper is set to 1.

condition and obtain a desired item j[6]. Once j is chosen, we update the model parameters by applying the SGD method.

We are interested in investigating whether the weight approximation yields the right contribution for the desired ranking loss. Based on Eq. (16), it is easy to achieve a potential loss induced by the violation of $\langle i,j\rangle_u$ pair using back-stepping approach

$$L\Big(\langle i,j\rangle_u\Big)=\frac{\sum_{r=0}^{\lceil\frac{|I|-1}{T}\rceil}\frac{1}{r+1}}{\Gamma(I)}\log\!\bigg(1{+}\exp\!\Big(-\sigma\big(\hat{y}(\boldsymbol{x}^i){-}\hat{y}(\boldsymbol{x}^j)\big)\Big)\bigg) \quad (17)$$

To simplify the function, we first omit the denominator term since it is a constant value, then we replace the CE loss with the 0/1 loss function.

$$\frac{\partial L(\langle i,j\rangle_u)}{\partial\theta}=\sum_{r=0}^{r(i)}\frac{1}{r+1}I\left[\hat{y}(\boldsymbol{x}^i)-\hat{y}(\boldsymbol{x}^j)\right] \quad (18)$$

where $I[\cdot]$ is the indicator function, $I[h]=1$ when h is true, and 0 otherwise. Thus, we have $I\left[\hat{y}(\boldsymbol{x}^i)-\hat{y}(\boldsymbol{x}^j\right]=1$ in all the cases because r is smaller than $r(i)$. Moreover, for each positive item i, there are $r(i)$ items that are ranked higher than i (i.e., item j), and thus each j has the probability of $p=\frac{1}{r(i)}$ to be picked. Finally, the formulation of the loss for each user u is simplified as[7]

$$L_u=\sum_{i\in I_u}\sum_{r=0}^{r(i)}\frac{1}{r+1} \quad (19)$$

As previously demonstrated in Section 3.2, the difference of pairwise losses between (b) and (c) in Figure 1 is inconsistent with that of NDCG values. Instead of using standard pairwise loss function, here we compute the losses based on Eq. (19), from which we achieve the losses of (b) and (c) are 2.93 and 4.43, respectively. This means, a smaller loss leads to a larger NDCG value, and vice versa. Similarly, we notice that the black (solid) arrows in (c) after movement lead to a larger loss than the red (dashed) ones, and in reason, the new NDCG value generated by the black ones is smaller than that by the red ones. Therefore, the movement direction and strength of red arrows is the right way to minimize a larger loss, which also demonstrates the correctness of the proposed lambda strategy. Based on Eq. (19), one can draw similar conclusions using other examples.

Regarding the properties of LFM-W, we observe that the weight approximation procedure is both dynamic (Line 9-13 of Algorithm 3) and context-aware (Line 11) during each update, similarly like LFM-D. Moreover, LFM-W is able to leverage both binary and graded relevance datasets (e.g. URL click numbers, POI check-ins), whereas LFM-S and LFM-D cannot learn the count information from positives. In terms of the computational complexity, using the gradient calculation in Eq. (16) can achieve important speedups. The complexity to compute $\triangle NDCG$ is $O(T_{\text{pred}}|I|)$ while the complexity with weighted approximation becomes $O(T_{\text{pred}}T)$. Generally, we have $T\ll|I|$ at the start of the training and $T<|I|$ when the training reaches a stable state. The reason is that at the beginning, LambdaFM is not well trained and thus it is quick to find an offending item j, which leads to a very small T, i.e., $T\ll|I|$, when the training converges to a stable state, most positive items are likely to be ranked correctly, which is also our expectation, and thus T becomes a bit larger. However, it is unlikely that all positive items are ranked correctly, so in general we have $T<|I|$.

[6]Item j is not limited to a non-positive item, since it can also be drawn from the positive collection I_u with a lower preference than i.

[7]Please note that L_u is non-continuous and indifferentiable, which means it is hard to be directly optimized with this form.

5. LAMBDA WITH ALTERNATIVE LOSSES

From Section 3.1, one may observe that the original lambda function was a specific proposal for the CE loss function used in RankNet [2]. To our knowledge, there exists a large class of pairwise loss functions in IR literature. The well-known pairwise loss functions, for example, can be margin ranking criterion, fidelity loss, exponential loss, and modified Huber loss, which are used in Ranking SVM [9], Frank [15], RankBoost [7], quadratically smoothed SVM [30], respectively. Motivated by the design ideas of these famous algorithms, we build a family of LambdaFM variants based on these loss functions and verify the generic properties of our lambda surrogates.

Margin ranking criterion (MRC):

$$L=\sum_{u\in U}\sum_{i\in I_u}\sum_{j\in I\setminus I_u}\max\Big(0,1-(\hat{y}(\boldsymbol{x}^i)-\hat{y}(\boldsymbol{x}^j))\Big) \quad (20)$$

MRC assigns each positive-negative (non-positive) item pair a cost if the score of non-positive item is larger or within a margin of 1 from the positive score. Optimizing this loss is equivalent to maximizing the area under the ROC curve (AUC). Since MRC is non-differentiable, we optimize its subgradient as follows

$$\lambda_{i,j}=\begin{cases}-1 & \text{if } \hat{y}(\boldsymbol{x}^i)-\hat{y}(\boldsymbol{x}^j)<1\\ 0 & \text{if } \hat{y}(\boldsymbol{x}^i)-\hat{y}(\boldsymbol{x}^j)\geq 1\end{cases} \quad (21)$$

As is known, the pairwise violations are independent of their positions in the ranking list [10]. For this reason, MRC might not optimize top-N very accurately.

Fidelity loss (FL): This loss is introduced by Tsai et al. [15] and has been applied in Information Retrieval (IR) task and yielded superior performance. The original function regarding the loss of pairs is defined as

$$L=\sum_{u\in U}\sum_{i\in I}\sum_{j\in I}\Big(1{-}\sqrt{\overline{P}_{ij}\cdot P_{i,j}}-\sqrt{(1-\overline{P}_{ij})\cdot(1-P_{i,j})}\Big) \quad (22)$$

where $\overline{P}_{ij}$ and P_{ij} share the same meanings with the CE loss in Eq. (1). According to the antisymmetry of pairwise ordering scheme [23], we simplify the FL in IFCAR settings as follows

$$L=\sum_{u\in U}\sum_{i\in I_u}\sum_{j\in I\setminus I_u}\left(1-\frac{1}{\sqrt{1+\exp\big(-\hat{y}(\boldsymbol{x}^i)+\hat{y}(\boldsymbol{x}^j)\big)}}\right) \quad (23)$$

In contrast with other losses (e.g., the CE and exponential losses), the FL of a pair is bounded between 0 and 1, which means the model trained by it is more robust against the influence of hard pairs. However, we argue that (1) FL is non-convex, which makes it difficult to optimize; (2) adding bound may cause insufficient penalties of informative pairs.

Modified Huber loss (MHL):

$$L=\sum_{u\in U}\sum_{i\in I_u}\sum_{j\in I\setminus I_u}\frac{1}{2\gamma}\max\Big(0,1-\big(\hat{y}(\boldsymbol{x}^i)-\hat{y}(\boldsymbol{x}^j)\big)\Big)^2 \quad (24)$$

where γ is a positive constant and is set to 2 for evaluation [30]. MHL is quadratically smoothed variant of MRC and proposed for linear prediction problems by Zhang et al. [30].

It is worth noting that both Fidelity and Huber loss functions have not been investigated yet in the context of recommendation. Besides, to the best of our knowledge, PRFM built on these two loss functions are novel. On the other hand, we find that exponential loss function is aggressive

Table 1: Basic statistics of datasets. Each entry indicates whether a user has interacted with an item.

Dataset	Users	Items	Artists	Albums	Entries
Yelp	10827	23115	-	-	333338
Lastfm	983	60000	25147	-	246853
Yahoo	2450	124346	9040	19851	911466

and seriously biased by hard pairs, which usually result in worse performance during the experiment. Thus the trial of PRFM with exponential loss have been omitted.

6. EXPERIMENTS

In this section, we conduct experiments on the three real-world datasets to verify the effectiveness of our proposed LambdaFM in various settings.

6.1 Experimental Setup

6.1.1 Datasets & Evaluation Metrics

We use three publicly accessible Collaborative Filtering (CF) datasets for our experiments: Yelp[8] (user-venue pairs), Lastfm[9] (user-music-artist triples) and Yahoo music[10] (user-music-artist-album tuples). To speed up the experiments, we follow the same procedures as in [5, 16] by randomly sampling a subset of users from the user pool of Yahoo datasets, and a subset of items from the item pool of Lastfm dataset. On Yelp dataset, we extract data from Phoenix and follow the common practice [23] to filter out users with less than 10 interactions. The reason is because the original Yelp dataset is much sparser than Lastfm and Yahoo datasets[11], which makes it difficult to evaluate recommendation algorithms (e.g., over half users have only one entry.). The statistics of the datasets after preprocessing are summarized in Table 1.

To illustrate the recommendation quality of LambdaFM, we adopt four standard ranking metrics: Precision@N and Recall@N (denoted by Pre@N and Rec@N respectively) [12], Normalized Discounted Cumulative Gain (NDCG) [16, 14] and Mean Reciprocal Rank (MRR) [26], and one (binary) classification metric, i.e. Area Under ROC Curve (AUC). For each evaluation metric, we first calculate the performance of each user from the testing set, and then obtain the average performance over all users. Due to the page limitations, we leave out the detailed formulas of these measures.

6.1.2 Baseline Methods

In our experiments, we compare our model with five powerful baseline methods[12]. **Most Popular (MP)** [16, 23]: It is to recommend users with the top-N most popular items. **User-based Collaborative Filtering (UCF)** [8, 23]: This is a typical memory-based CF technique for recommender systems. The preference quantity of user u to a candidate item i is calculated as an aggregation of some similar users' preference on this item. Pearson correlation is used in our work to compute user similarity and the top-30 most similar users are selected as the nearest neighbors. **Factorization Machines (FM)** [20, 21]: FM is designed for the rating prediction task (known as a pointwise approach). Here we adapt FM for the item recommendation task by binarizing the rating value. Note that, to conduct a fair comparison, we also develop the bootstrap sampling to make use of non-positive items, which outperforms FM in libFM [21]. **Bayesian Personalized Ranking (BPR)** [23]: This is one of the strongest context-free recommendation algorithms specifically designed for top-N item recommendations based on implicit feedback. **Pairwise Ranking Factorization Machines (PRFM)** [18]: To the best of our knowledge, PRFM is the state-of-the-art context-aware ranking algorithm, optimized to maximize the AUC metric. In this paper, we develop several PRFM[13] algorithms by applying various pairwise loss functions introduced in Section 5.

6.1.3 Hyper-parameter Settings

Learning rate η: We apply the 5-fold cross validation to find η for PRFM[14], and then employ the same η to LambdaFM for comparison. In addition, for BPR, the experimental results show that it performs the best with the same set of parameters of PRFM; for FM, we apply the same procedure to chose η independently. **Latent dimension** d: The effect of latent factors has been well studied by previous work, e.g., [23]. For comparison purposes, the approaches (e.g., [16]) are usually to assign a fixed d value (e.g., $d = 30$ in our experiments) for all methods based on factorization models. **Regularization** γ_θ: LambdaFM has several regularization parameters, including γ_{w_k}, $\gamma_{v_{k,f}}$, which represent the regularization parameters of latent factor vectors of w_k, $v_{k,f}$, respectively. We borrow the idea from [21] by grouping them for each factor layer, i.e., $\gamma_\pi = \gamma_{w_k}$, $\gamma_\xi = \gamma_{v_{k,f}}$. We run LambdaFM with $\gamma_\pi, \gamma_\xi \in \{0.5, 0.1, 0.05, 0.01, 0.005\}$ to find the best performance parameters. **Distribution coefficient** ρ: $\rho \in (0, 1]$ is specific for LFM-S and LFM-D, which is usually tuned according to the data distribution.

6.2 Performance Evaluation

All experiments are conducted with the standard 5-fold cross validation. The average results over 5 folds are reported as the final performance.

6.2.1 Accuracy Summary

Table 2 and Figure 2(a-f) show the performance of all the methods on the three datasets. Several insightful observations can be made: First, in most cases personalized models (UCF, BPR, PRFM, LambdaFM) noticeably outperform MP, which is a non-personalized method. This implies, in practice, when there is personalized information present, personalized recommenders are supposed to outperform non-personalized ones. Particularly, our LambdaFM clearly outperforms all the counterparts in terms of four ranking metrics. Second, we observe that different models yield basically consistent recommendation accuracy on different metrics, except for AUC, which we give more details later. Third, as N increases, values of Pre@N get lower and values of Rec@N become higher. The trends reveal the typical behavior of recommender systems: the more items are recommended, the better the recall but the worse the precision achieved.

Regarding the effectiveness of factorization models, we find that BPR performs better than FM on the Yelp dataset[15],

[8] https://www.yelp.co.uk/dataset_challenge

[9] http://www.dtic.upf.edu/~ocelma/MusicRecommendationDataset/lastfm-1K.html

[10] webscope.sandbox.yahoo.com/catalog.php?datatype=r&did=2

[11] Cold users of both datasets have been trimmed by official provider.

[12] For the sake of understanding the ranking effectiveness of different algorithms, we launch a random one (denoted as Rand) as in [8].

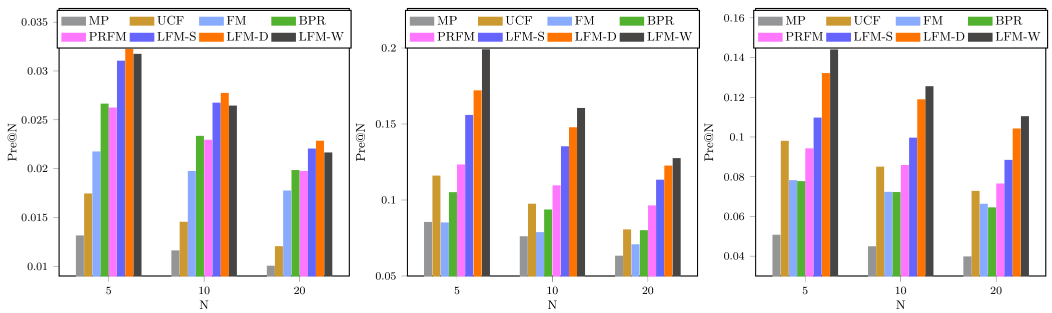
[13] PRFM is short for PRFM with the CE loss if not explicitly declared.

[14] η is set to 0.01 on Yelp and Yahoo datasets, and 0.08 on Lastfm.

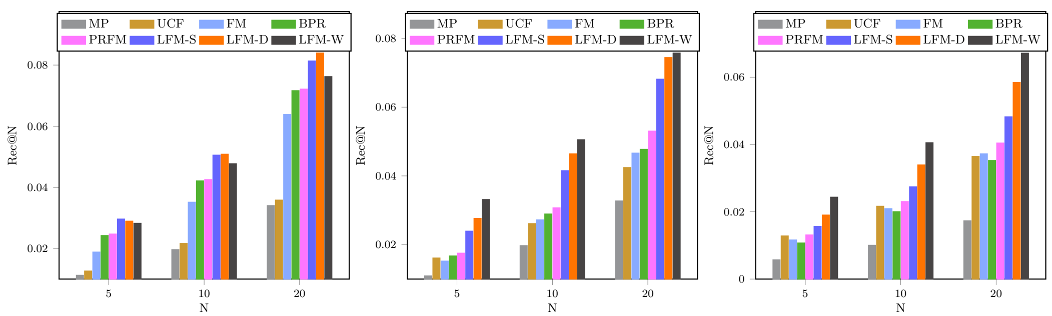
[15] Note that it is not comparable on the Lastfm and Yahoo datasets since FM combines other contexts.

Table 2: Performance comparison on NDCG, MRR and AUC, where "*" means significant improvement in terms of paired t-test with p-value < 0.01. For each measure, the best result is indicted in bold.

Dataset	Metrics	Rand	MP	UCF	FM	BPR	PRFM	LFM-S	LFM-D	LFM-W	Improv.
Yelp	NDCG	0.1279	0.1802	0.1714	0.2130	0.2186	0.2186	0.2218	**0.2232**	0.2191	+2.10%*
	MRR	0.0019	0.0451	0.0557	0.0718	0.0860	0.0852	0.0950	**0.0997**	0.0977	+15.93%*
	AUC	0.5007	0.8323	0.6203	0.8981	0.9043	**0.9044**	0.8876	0.8787	0.874	-
Lastfm	NDCG	0.2330	0.3452	0.3449	0.3832	0.3830	0.3944	0.4095	0.4175	**0.4191**	+6.26%*
	MRR	0.0057	0.2051	0.2634	0.2182	0.2588	0.2856	0.3433	0.3514	**0.3914**	+37.04%*
	AUC	0.4987	0.8506	0.6661	0.9161	0.9055	**0.9209**	0.9145	0.9075	0.8949	-
Yahoo	NDCG	0.2232	0.3109	0.3362	0.3682	0.3478	0.3720	0.3791	0.3993	**0.4016**	+7.96%*
	MRR	0.0039	0.1252	0.2219	0.1942	0.1909	0.2211	0.2467	0.2857	**0.3004**	+35.87%*
	AUC	0.5005	0.8425	0.7491	0.9313	0.8720	**0.9357**	0.9256	0.9340	0.9273	-

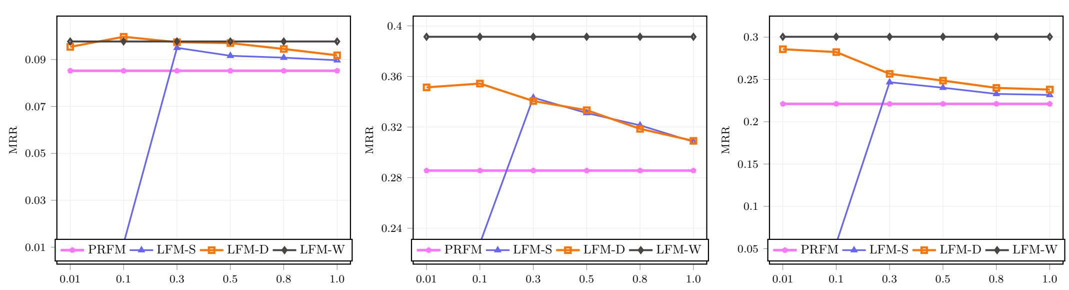


(a) P-Yelp **(b)** P-Lastfm **(c)** P-Yahoo



(d) R-Yelp **(e)** R-Lastfm **(f)** R-Yahoo

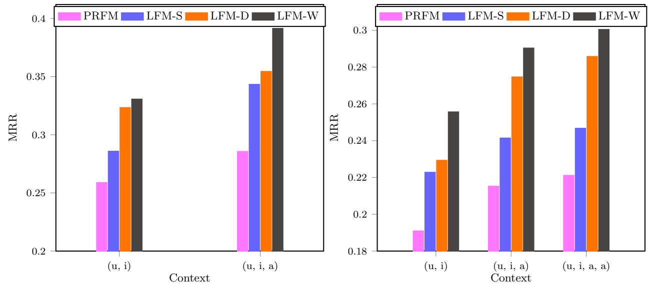
Figure 2: Performance comparison with respect to top-N values, i.e., Pre@N (P) and Rec@N (R).



(a) Yelp **(b)** Lastfm **(c)** Yahoo

Figure 3: Parameter tuning with respect to MRR.

which empirically implies pairwise methods outperform pointwise ones for ranking tasks. The reason is that FM is identical to matrix factorization with only user-item information. In this case, the main difference of FM and BPR is that they apply different loss functions, i.e., quadratic and logistic loss, respectively. This is in accordance with the interesting finding that BPR and PRFM produce nearly the same results w.r.t. all metrics on Yelp. Furthermore, among all the baseline models, the performance of PRFM is very promising. The reason is that PRFM, (1) as a ranking-based factorization method, is more appropriate for handling top-N item recommendation tasks (vs. FM); (2) by applying FM as scoring function, PRFM estimates more accurate ordering relations than BPR due to auxiliary contextual variables.



(a) Lastfm **(b)** Yahoo

Figure 4: Performance comparison in terms of MRR with different context.

LFM vs. PRFM: We observe that in Figure 2 and Table 2 our LambdaFM (including LFM-S, LFM-D and LFM-W) consistently outperforms the state-of-the-art method PRFM in terms of four ranking metrics. In particular, the improvements on Lastfm and Yahoo datasets w.r.t. NDCG and MRR, are more than 6% and 35%, respectively. Similar improvements w.r.t. Pre@N and Rec@N metrics can be observed from Figure 2[16]. Besides, we observe that LFM underperforms PRFM in terms of AUC. The results validate our previous analysis, namely, LambdaFM is trained to optimize ranking measures while PRFM aims to maximize the AUC metric. This indicates that pairwise ranking models, such as BPR and PRFM, may not be good approximators of the ranking biased metrics and such a mismatch empirically leads to non-optimal ranking.

6.2.2 *Effect of Lambda Surrogates*

In this subsection, we study the effect of lambda surrogates and parameter influence on the recommendation performance. For LFM-S, we directly tune the value of ρ; for LFM-D, we fix the number of sampling units m to a constant, e.g., $m = 10$, and then tune the value of $\rho \in \{0.01, 0.1, 0.3, 0.5, 0.8, 1.0\}$; and for LFM-W the only parameter ε is fixed at 1 for all three datasets in this work. Figure 3 depicts the performance changes by tuning ρ. First, we see LFM-W performs noticeably better than PRFM, and by choosing a suitable ρ (e.g., 0.3 for LFM-S and 0.1 for LFM-D), both LFM-D and LFM-S also perform largely better than PRFM. This indicates the three suggested lambda surrogates work effectively for handling the mismatch drawback between PRFM and ranking measures. Second, LFM-S produces best accuracy when setting ρ to 0.3 on all datasets but the performance experiences a significant decrease when setting ρ to 0.1. The reason is that the SGD learner will make more gradient steps on popular items due to the oversampling when ρ is set too small according to Eq. (12). In this case, most less popular items will not be picked for training, and thus the model is under-trained. By contrast, the performance of LFM-D has not dropped on Yahoo dataset even when ρ is set to 0.01 (which means picking the top from m randomly selected items, see Algorithm 2). This implies, the performance may be further improved by setting a larger m. Third, the results indicate that LFM-D and LFM-W outperform LFM-S on Lastfm and Yahoo datasets[17]. This is because the static sampling method ignores the fact that the estimated ranking of a non-positive item j changes during learning, i.e., j might be ranked high in the first step but it is ranked low after several iterations[18]. Besides, LFM-S computes the item ranking based on the overall popularity, which does not reflect the context information. Thus, it is more encouraged to exploit current context for computing ranking such as LFM-D and LFM-W.

6.2.3 *Effect of Context*

Finding competitive contextual features is not the main focus of this work but it is interesting to see to what extent LambdaFM improves the performance by adding context. Therefore, we conduct a contrast experimentation and show the results on Figure 4(a-b)[19], where (u, i) denotes a user-item (i.e., music) pair and (u, i, a) denotes a user-item-artist triple on both datasets; similarly, (u, i, a, a) represents a user-item-artist-album quad on Yahoo dataset. First, we observe LambdaFM performs much better with (u, i, a) tuples than that with (u, i) tuples on both datasets. This result is consistent with the intuition that a user may like a song if she likes another song by the same artist. Second, as expected, LambdaFM with (u, i, a, a) tuples performs further better than that with (u, i, a) tuples from Figure 4(b). We draw the conclusion that, by inheriting the advantage of FM, the more effective context incorporated, the better LambdaFM performs.

6.2.4 *Lambda with Alternative Loss Functions*

Following the implementations of well-known pairwise LtR approaches introduced in Section 5, we have built a family of PRFM and LambdaFM variants. Figure 5 shows the performance of all these variants based on corresponding loss functions. Based on the results, we observe that by implementing the lambda surrogates, all variants of LambdaFM outperform PRFM, indicating the generic properties of these surrogates. Among the three lambda surrogates, the performance trends are in accordance with previous analysis, i.e. LFM-W and LFM-D perform better than LFM-S. Additionally, most recent literature using pairwise LtR for recommendation is based on BPR optimization criterion, which is equivalent to the CE loss in the implicit feedback scenarios. To the best of our knowledge, the performance of FL and MHL loss functions have not been investigated in the context of recommendation, and also PRFM and LambdaFM based on FL and MHL loss functions achieve competitive performance with the ones using the CE loss. We expect our suggested PRFM (with new loss functions) and LambdaFM to be valuable for existing recommender systems that are based on pairwise LtR approaches.

7. CONCLUSION AND FUTURE WORK

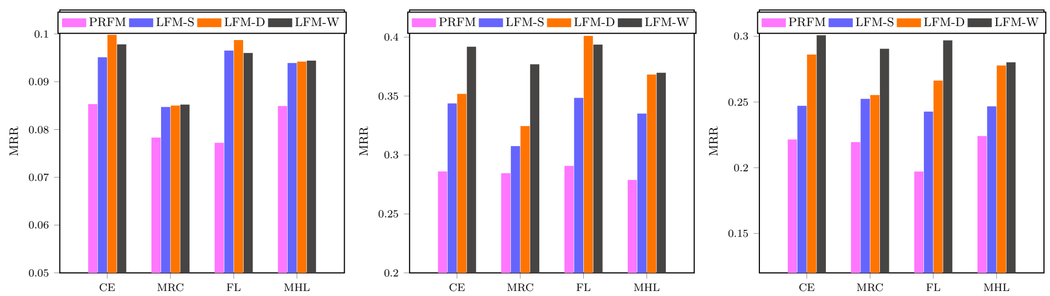
In this paper, we have presented a novel ranking predictor Lambda Factorization Machines (LambdaFM). Inheriting advantages from both LtR and FM, LambdaFM (i) is capable of optimizing various top-N item ranking metrics in implicit feedback settings; (ii) is very flexible to incorporate context information for context-aware recommendations. Different from the original lambda strategy, which is tailored for the CE loss function, we have proved that our proposed lambda surrogates are more general and applicable

[16]To save space, we only use MRR for the following discussion since the performance trend on other ranking metrics is consistent.

[17]The superiority is not obvious on the Yelp dataset. This might be due to the reason that the Yelp dataset has no additional context and the item tail is relatively shorter than that of other datasets.

[18]In spite of this, non-positive items drawn by popularity sampler are still more informative than those drawn by uniform sampler.

[19]ρ is assigned a fixed value for the study of context effect, namely 0.1 and 0.3 for LFM-D and LFM-S, respectively.



(a) Yelp **(b)** Lastfm **(c)** Yahoo

Figure 5: The variants of PRFM and LambdaFM based on various pairwise loss functions.

to a set of well-known ranking loss functions. Furthermore, we have built a family of PRFM and LambdaFM algorithms, shedding light on how they perform in real tasks. In our evaluation, we have shown that LambdaFM largely outperforms state-of-the-art counterparts in terms of four standard ranking measures, but underperforms PRFM methods in terms of AUC, known as a binary classification measure.

LambdaFM is also applicable to other important ranking tasks based on implicit feedback, e.g., personalized microblog retrieval and learning to personalize query autocompletion. In future work, it would be interesting to investigate its effectiveness in these scenarios.

8. ACKNOWLEDGMENTS

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